Pearson Edexcel

Model answer- checked

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Applied) (Public release version)

Resource Set 1: Topic 7

Kinematics (Test 1)

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}

(*a*) Find the velocity of P at time $t = 2$ seconds.

(2)

a) $\underline{v} = \underline{u} + \underline{a}t$

$$\underline{v} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -8 \end{pmatrix} \text{ ms}^{-1}$$

$$= (6\underline{i} - 8\underline{j}) \text{ ms}^{-1}$$

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

Find the value of T.

(4)

b) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\begin{pmatrix} \lambda \\ -4.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + T\begin{pmatrix} -2 \\ 2 \end{pmatrix} + \frac{T^2}{2}\begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$-4.5 = 2T - 2.5T^2 \Rightarrow 2.5T^2 - 2T - 4.5 = 0$

$\Rightarrow T = 1.8\text{s} \quad T = -1\text{s}$

$\Rightarrow T = 1.8\text{s} \quad (T > 0)$

Hence find the value of λ

c) $\lambda = 0 + 1.8(-2) + \frac{1}{2}(1.8)^2(4)$

$= 2.88$

(2)

(Total for Question 1 is 8 marks)

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.] A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j}) \text{ m s}^{-1}$.

The acceleration of the boat is constant.

Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$.

(2)

a) $\underline{v} = \underline{u} + \underline{a}t$

$\begin{pmatrix} 10.5 \\ -0.9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.6 \end{pmatrix} + 15\underline{a} \implies$ $10.5 = 15a_x \implies a_x = 0.7$; $-0.9 = 0.6 + 15a_y \implies a_y = \frac{-1.5}{15} = -0.1$

$\underline{a} = \begin{pmatrix} 0.7 \\ -0.1 \end{pmatrix} \text{ms}^{-2} = (0.7\underline{i} - 0.1\underline{j}) \text{ms}^{-2}$

Find **r** in terms of t.

(2)

b) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0.6 \end{pmatrix} + \frac{t^2}{2}\begin{pmatrix} 0.7 \\ -0.1 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 0.35t^2 \\ -0.05t^2 + 0.6t \end{pmatrix}$ $\quad \underline{r} = (0.35t^2\underline{i} - (0.05t^2 + 0.6t)\underline{j})\text{ m}$

Find the value of t when the boat is north-east of O.

(3)

c) North-east $\implies$ i and j components of $\underline{r}$ are equal

So $0.35t^2 = -0.05t^2 + 0.6t$

$0.4t^2 = 0.6t$

$4t^2 - 6t = 0$

$2t(2t-3) = 0 \implies t = 0, \; t = 1.5\text{s}$

nd the value of t when the boat is moving in a north-east direction. **(3)**

d) Moving north-east => i and j components of $\underline{v}$ are equal

$$\underline{v} = \underline{u} + \underline{a}t$$

$$= \begin{pmatrix} 0 \\ 0.6 \end{pmatrix} + t\begin{pmatrix} 0.7 \\ -0.1 \end{pmatrix} = \begin{pmatrix} 0.7t \\ -0.1t + 0.6 \end{pmatrix}$$

$$0.7t = -0.1t + 0.6$$

$$0.8t = 0.6 \Rightarrow t = 0.75\text{s}$$

(Total for Question 2 is 10 marks)

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3. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration. At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m.

(a) Show that the magnitude of the acceleration of P is 2.5 m s^{-2}.

(4)

a) $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\begin{pmatrix} 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -3 \end{pmatrix} + 2\underline{a}$$

$$\begin{pmatrix} 3 \\ -4 \end{pmatrix} = 2\underline{a} \Rightarrow \underline{a} = \begin{pmatrix} 1.5 \\ -2 \end{pmatrix}$$

$$|\underline{a}| = \sqrt{1.5^2 + (-2)^2} = 2.5\text{ms}^{-2}$$

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})$ m s^{-2}.

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

(Total for Question 3 is 8 marks)

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b) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + 2\begin{pmatrix} 1.5 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$: velocity at A

$\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} + t\begin{pmatrix} 4 \\ 8.8 \end{pmatrix} \Rightarrow 5 + 4t = -7 + 8.8t$

$12 = 4.8t$

$t = 2.5$ seconds

4. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\ \mathrm{m\ s^{-2}}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\ \mathrm{m\ s^{-1}}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(*a*) Find the value of T.

(4)

a) $\underline{v} = \underline{u} + \underline{a}t$

$$\begin{pmatrix}3k\\-4k\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix} + T\begin{pmatrix}2\\-3\end{pmatrix}$$

$3k+1 = 2T \Rightarrow 3k - 2T = -1$ $\Rightarrow k = 5,\ T = 8$

$-4k-4 = -3T \Rightarrow -4k + 3T = 4$ (from calculator)

$T = 8\ \mathrm{s}$

(*b*) Find the distance AB.

b) $S = ut + \frac{1}{2}at^2$

$$= 4\begin{pmatrix}-1\\4\end{pmatrix} + \frac{1}{2}(4)^2\begin{pmatrix}2\\-3\end{pmatrix}$$

$$= \begin{pmatrix}-4\\16\end{pmatrix} + 8\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}12\\-8\end{pmatrix}$$

$|S| = \sqrt{12^2 + (-8)^2}$

$= 4\sqrt{13}\ \mathrm{m} = 14.4\ \mathrm{m}$

(Total for Question 2 is 8 marks)

5.

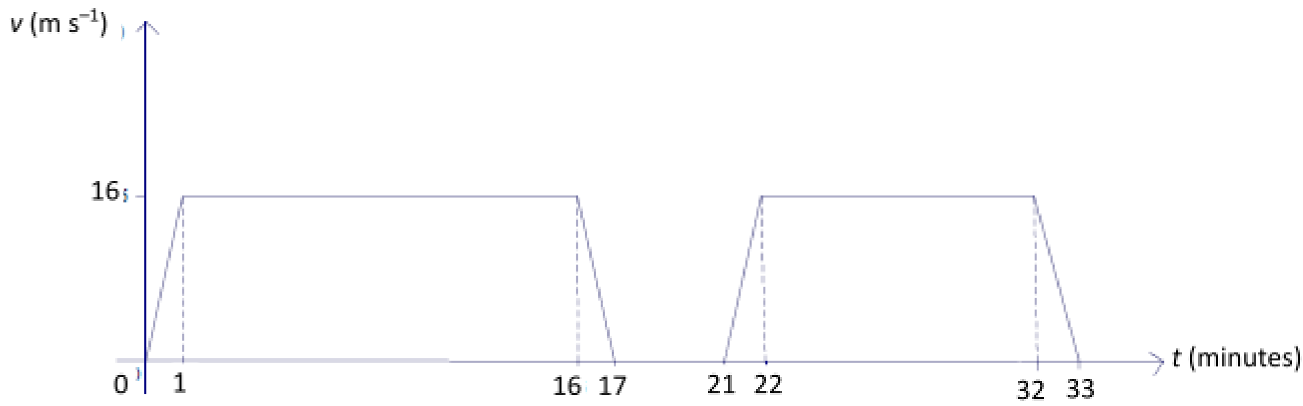


Figure 2

A train X runs on a straight horizontal track that connects stations A and C. Station B lies between A and C. Figure 2 shows the graph of the speed, v m s^{-1}, of train X against the time, t minutes, after noon.

Train X leaves A at noon and accelerates uniformly from rest until $t = 1$, when it is moving with speed 16 m s^{-1}.

Train X then continues to move along the track at constant speed for 15 minutes, before decelerating uniformly and coming to rest at B at time $t = 17$.

Train X leaves B at time $t = 21$and accelerates uniformly for one minute, reaching a speed of 16 m s^{-1}.

Train X then moves along the track at a constant speed of 16 m s^{-1} for 10 minutes, before decelerating uniformly and coming to rest at C at time $t = 33$.

(a) Find the distance of C from A, stating the units of your answer.

(3)

a) Area represents distance

Area $= 2\left(16 \times \frac{1}{2} + 15 \times 16 + 16 \times \frac{1}{2}\right) = 512$

$512 \times 60 = 30{,}720$ m (the x-axis in in minutes)

A second train, Y, leaves A at T minutes after noon and moves in the same direction as train X on a parallel straight horizontal track.

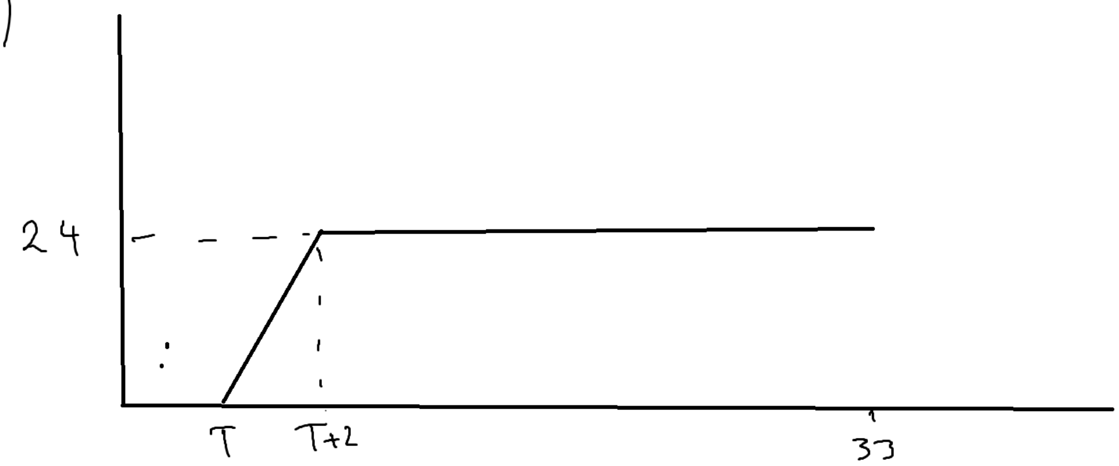
Train Y accelerates uniformly from rest for 2 minutes, reaching a speed of 24 m s^{-1}.

Train Y then moves along the track at a constant speed of 24 m s^{-1} and passes C at this speed.

Train Y passes C at the instant train X stops at C.

(b) Find the value of T. **(5)**

b)



$$\text{Area} = \frac{24 \times 2}{2} + (33 - T - 2) \times 24 = 768 - 24T$$

$$60(768 - 24T) = 30720$$

$$768 - 24T = \frac{30720}{60}$$

$$768 - \frac{30720}{60} = 24T$$

$$\Rightarrow T = 10.67 \text{ minutes}$$

(c) State one assumption made in your working that could affect the accuracy of your answer to part (b).

c) That the difference in resistance of the different tracks is negligible

(Total for Question 5 is 9 marks)

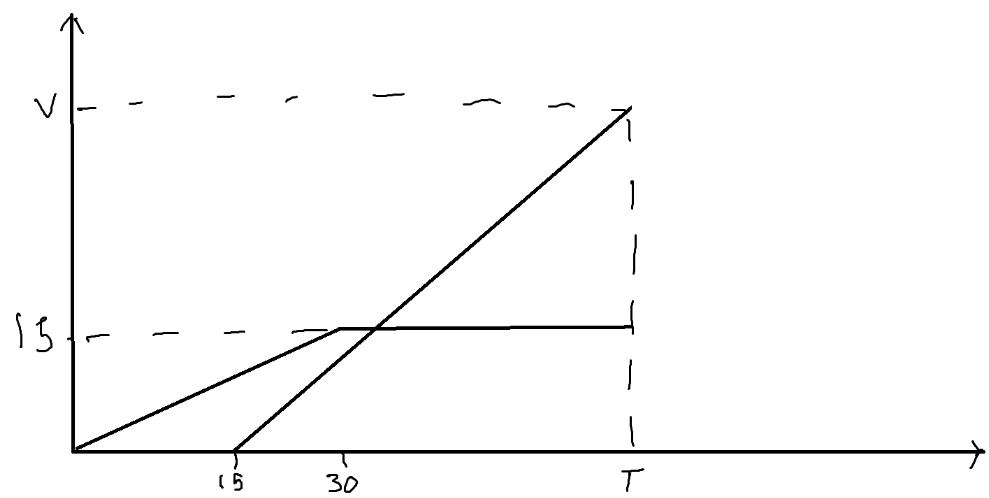
6. A car moves along a straight horizontal road. The car starts from rest at a fixed point A on the road and moves with constant acceleration for 30 seconds, reaching a speed of 15 m s^{-1}. This speed is then maintained.

When the car has been moving for 15 seconds a motorbike starts from rest at A and moves along the same road in the same direction as the car. The motorbike accelerates at 1.5 m s^{-2} so that it catches up with the car when the car has been moving for T seconds.

(a) Using the same axes, sketch the speed-time graph of the car and the speed-time graph of the motorbike up to the time when the motorbike catches up with the car.

(3)

a)



(b) Find the speed of the motorbike at the instant it catches up with the car.

(6)

b) Distances of the car and motorbike are equal

$$\frac{15 \times 30}{2} + 15(T-30) = \frac{V(T-15)}{2}$$

$$15T - 225 = \frac{V}{2}(T-15)$$

$$15(T-15) = \frac{V}{2}(T-15)$$

$$15 = \frac{V}{2} \Rightarrow V = 30 \text{ ms}^{-1}$$

(Total for Question 6 is 9 marks)